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QUESTIONS OF SIMILARITY AND THE SCATTERING OF WAVES IN VISCOPLASTIC MEDIA

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A study of plane waves in viscous media was made in [1-7]. A solution of the problem of the propagation of a wave set up by unsteady-state shock loading in a viscoelastic medium was obtained using an electronic computer in [6], and a solution in a viscoplastic medium in [1, 7]. In the latter case, different equations are introduced describing the behavior of the medium with loading and unloading, which leads to the formation of residual deformations. On the basis of the solutions of [1, 7], a finite-difference representation was constructed for the equations of motion in Lagrange variables, and for the sequence of differential equations determining the behavior of the medium. The method of "straight-through" calculation with pseudoviscosity was used. The introduction of the pseudoviscosity brings about the replacement of the shock fronts by regions of a continuous change in the parameters, which leads to additional difficulties in determination of the laws governing the washing-out of a shock wave and the scattering of waves. Below, the method of characteristic curves is used to obtain a solution to the problem of the propagation of a plane wave, set up by an unsteady-state shock load in a linear viscoplastic medium, corresponding to the model of [1]. It follows from the calculations that volumetric viscosity leads to scattering of the waves and to nonobservance of the condition of similarity. An increase by an order of magnitude in duration of a wave changes the rate of propagation of the maximum of the stresses, and the stresses themselves, by only a few percent. The values of the deformation and the velocity of the particles vary to a greater de-

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gree than the stress. It has been shown that, in the medium, there generally arises a double-wave configuration. In front, there moves a forerunner, with a shock wave at the front. In the vicinity of the initial cross section, behind the shock wave there follows a decrease, and then a continuous rise in the stress to a second maximum, followed by a decrease in the stress. At a sufficient distance behind the shock wave, there is a continuous rise in the stress to a maximum, and then a decrease in it. At still greater distances, the amplitude of the shock wave is practically equal to zero, and the stress rises and falls continuously. The model of [1] is designed for a description of soils and rocks, as well as some other solid media.

1. Statement of Problem, Method of Solution

We use a model of a viscoplastic medium [1], in accordance with which, in a medium, there exist dynamic $\sigma = E_D \varepsilon_1$ (with $\dot{\varepsilon} \to \infty$) and static $\sigma = E_S \varepsilon$ (with $\dot{\varepsilon} \to 0$) diagrams of the compression. Unloading takes place in accordance with different equations than loading, which leads to the formation of residual deformations and reflects the plastic properties of the medium.

The deformation of an element of the medium has the form

$$\varepsilon = \varepsilon_1 + \varepsilon_2,$$

where ε_1 is connected with the instantaneous (dynamic) compression of the material, and ε_2 with the repacking of the grains, taking place in the course of a finite period of time. With a decrease in the load, ε_1 varies according to the law

$$\sigma - \sigma_m = E_R(\varepsilon_1 - \varepsilon_m),\ \varepsilon_m = \sigma_m / E_D,\ E_R > E_D.$$

The deformation ε_2 is assumed to be irreversible. Under these assumptions, the behavior of the medium is determined by the following equations:

a) with shock loading,

$$\varepsilon = \varepsilon_1 = \sigma / E_D; \tag{1}$$

b) with a continuous rise in the stress,

$$\dot{\varepsilon} + \mu\varepsilon = \dot{\sigma}/E_D + \mu\sigma/E_S;$$
$$\mu = E_D E_S / (E_D - E_S)\eta; \tag{2}$$

c) with a decrease in the stress, but a rise in ε_2,

$$\dot{\varepsilon} + \mu\varepsilon = \dot{\sigma}/E_R + \mu\sigma\left(E_S^{-1} - E_D^{-1} - E_R^{-1}\right) + \mu\sigma_m\left(E_D^{-1} - E_R^{-1}\right); \tag{3}$$

d) with a decrease in the stress and $\varepsilon_2 = \text{const}$,

$$E_R \dot{\varepsilon} = \dot{\sigma}, \tag{4}$$

where E_D is the dynamic compression modulus; E_S is the static compression modulus; E_R is the modulus of the unloading; η is the coefficient of the viscosity; μ is the parameter of the viscosity.

We use Lagrangian variables (h is the mass, t is the time). The load in the initial cross section $h = 0$, setting up a wave (the first boundary condition) is represented by the expressions

$$\sigma = \sigma_m(1 - t/\theta),\ 0 \leqslant t \leqslant \theta,\ \sigma = 0,\ t \geqslant \theta. \tag{5}$$

We assume $\gamma = E_D/E_S$, $\beta = E_D/E_R$; the acoustic (wave) resistance (impedance) of the medium $A = \sqrt{E_D \rho_0} = c_0 \rho_0$, where ρ_0 is the initial density of the medium; c_0 is the speed of the sound of the longitudinal wave. We write the equation of the line of the front in the plane h, t:

$$h = At.$$

Then the second boundary condition (at the front of the wave, where viscous properties do not appear) can be represented by the expression

$$\sigma = -Au \text{ where } h = At. \tag{6}$$

We go over to the dimensionless variables

$$\tau = \mu t;\quad x = \mu h / A;\quad \sigma^0 = \sigma/\sigma_m;\quad u^0 = u/u_m;\quad \varepsilon^0 = \varepsilon/\varepsilon_m;\quad u_m = -\sigma_m A;$$
$$\varepsilon_m = \sigma_m / E_D.$$

TABLE 1

No. of variant	1	2	3	4	5	6
Value of $\mu\theta$	50000	500	50	10	5	2,5

The basic equations of motion in these variables have the form

$$\frac{\partial u^0}{\partial \tau}+\frac{\partial \sigma^0}{\partial x}=0;\quad \frac{\partial u^0}{\partial x}+\frac{\partial \varepsilon^0}{\partial \tau}=0$$

and Eqs. (1)-(4) $\sigma^0=\varepsilon^0$; $\dot{\varepsilon}^0+\varepsilon^0=\sigma^0+\gamma\sigma^0$;

$$\dot{\varepsilon}^0+\varepsilon^0=\beta\dot{\sigma}^0+\sigma^0(\gamma+\beta-1)+\sigma^0_{\max}(1-\beta);\ \dot{\varepsilon}^0=\beta\dot{\sigma}^0,$$

where $\sigma^0_{\max}$ is the maximal dimensionless stress in a particle of the medium. Then the boundary condition (5) (x = 0) is written as $\sigma^0=1-\tau/\mu\theta$, $0\le\tau\le\lambda$; $\sigma^0=0$, $\tau\ge\mu\theta$, and (6) ($x=\tau$),

$$\sigma^0=u^0.$$

The system of equations is hyperbolic. The characteristic relationships in the plane x, τ have the form

$$d\sigma^0\pm du^0=(\varepsilon^0-\gamma\sigma^0)d\tau \text{ with } dx/d\tau=\pm 1;$$
$$d\sigma^0-d\varepsilon^0=(\varepsilon^0-\gamma\sigma^0)d\tau \text{ with } dx/d\tau=0$$

in the region where $\dot{\sigma}>0$, $\dot{\varepsilon}_2>0$, which corresponds to the condition $d\sigma^0/d\tau>0$, $d\sigma^0/d\tau<d\varepsilon^0/d\tau$;

$$d\sigma^0\pm\frac{du^0}{\sqrt{\beta}}=\frac{1}{\beta}\left[\varepsilon^0-(\gamma+\beta-1)\sigma^0-(1-\beta)\sigma^0_m\right]d\tau$$

with $dx/d\tau=\pm\beta^{-1/2}$,

$$d\sigma^0-\frac{1}{\beta}d\varepsilon^0=\frac{1}{\beta}\left[\varepsilon^0-(\gamma+\beta-1)\sigma^0-(1-\beta)\sigma^0_m\right]d\tau$$

with $dx/d\tau=0$ in the region where $\dot{\sigma}<0$, $\dot{\varepsilon}_2>0$, which corresponds to the condition $d\sigma^0/d\tau<0$, $d\sigma^0/d\tau<\beta d\varepsilon^0/d\tau$;

$$\sqrt{\beta}d\sigma^0\pm du^0=0 \text{ with } dx/d\tau=\pm 1/\sqrt{\beta};$$
$$\beta d\sigma^0-d\varepsilon^0=0 \text{ with } dx/d\tau=0$$

in the region where $\dot{\sigma}<0$, $\dot{\varepsilon}_2=0$, which corresponds to the condition $d\sigma^0/d\tau<0$, $d\sigma^0/d\tau=\beta d\varepsilon^0/d\tau$.

2. Results of the Solution and Their Analysis

Six variants were calculated in an Odra digital computer. In all cases it was assumed that $\gamma=2$, $\beta=0.5$. The values of $\mu\theta$ are given in Table 1.

The results of a calculation of the parameters of a wave at fixed points of the medium for $\mu\theta=5$ are given in Fig. 1 (a is the stress, b is the deformation, and c is the velocity of the particles). The distances considered are sufficiently far from the origin of coordinates. Curves 0-4 illustrate distances x from the initial cross section equal to 0, 5, 10, 20, and 40, respectively. At all distances, a forerunner moves in front, at whose front all the parameters vary discontinuously. The value of the discontinuity does not depend on the value of $\mu\theta$. It falls rapidly with increasing distance. The small circles denote parameters at the forerunner, attained by the discontinuity. With x = 5, the value of the discontinuity is equal to 0.079, and, with x = 10, to only 0.0063. In a viscoelastic medium, where loading and unloading of the medium take place in accordance with exactly the same equation, with $\gamma=2$ and $\beta=1$ the values of the discontinuity at these same distances are equal, respectively, to 0.082 and 0.007 [6]. Behind the shock wave, there is a continuous rise in the parameters up to a maximum, followed by a decrease. With increasing distance from the initial cross section, the values of all the parameters decrease.

Calculations show that, with an increase in $\mu\theta$, the rate of damping of the wave decreases with increasing distance, while the time required to attain a maximum rises. With $\mu\theta=50{,}000$, the wave is close to stationary; at the distances under consideration, the values of the parameters practically do not vary, and the time required to attain a maximum is the greatest. With $\mu\theta=0.5$, the wave lags behind the shock wave and there is no washing-out [6].

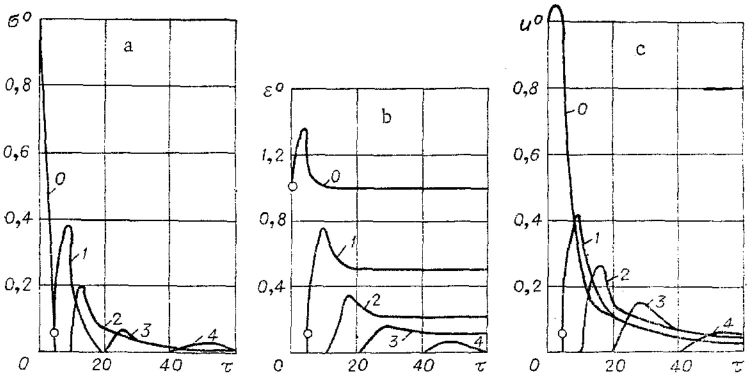

Fig. 1

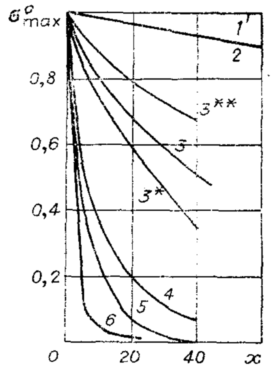

Fig. 2

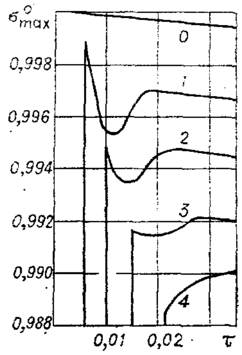

Fig. 3

In the initial cross section and near it, there is first attained a maximum of the stress, and then of the deformation and the velocity of the particles. With increasing distance from the initial cross section, the moments of the attainment of a maximum of all the parameters approach each other and, at a sufficiently great distance, are practically identical. With the passage of time, the deformation tends toward a constant (residual) value, which depends on the distance and the value of $\mu\theta$.

Figure 2 gives curves of the dependence of the maximal stress on the distance, with different values of $\mu\theta$. Here and in what follows, the numbering of the curves corresponds to the number of the variants given in Table 1. Curves 3* and 3** were plotted from the data of [7], obtained with $\mu\theta=5$, $\beta=0.5$, and γ equal, respectively, to 4 and 1.1. With a rise in γ, there is an increase in the difference between the diagrams of the dynamic and static compression. This leads to a rise in the losses of energy in the wave, and to its more rapid damping with increasing distance. Curve 3**, therefore, lies above, and 3* below, curve 3.

The calculations show that, with increasing distance, the rate of damping of the maximal value of the deformation (ε^0_{max}), like the stress, depends essentially on $\mu\theta$.

With $\mu\theta=50{,}000$, the maximal deformation is practically equal to the limiting value attained on the diagram of the static compression, and the residual deformation corresponds to unloading, from the static diagram of the compression: $\varepsilon^0_m \sim 1.99$, $\varepsilon^0_{res} \sim 1.50$. With a decrease in $\mu\theta$, the maximal and residual deformations decrease. With $\mu\theta = 5$, at a sufficiently great distance, they approximately are two orders of magnitude less than with $\mu\theta=50{,}000$.

Let us consider in more detail the change in the stress at fixed points of the medium near the initial cross section. In the region between the front of the wave (the forerunner) and the initial cross section, the solution is determined by continuously differentiable functions. With $x=0$, in some neighborhood of the initial cross section, behind the shock wave $d\sigma^0/d\tau < 0$. Figure 3 shows values of $\sigma^0(\tau)$ in the neighborhood of the initial cross section with $\mu\theta=50$ (variant 3). Curves 0-4 correspond to distances x equal to 0, 0.007, 0.01, 0.015, and 0.02, respectively. It can be seen from the curves that, in the case of a wave, set up by an unsteady-state shock loading, there are two maxima near the initial cross section. With propagation of the wave, the first maximum is rapidly smoothed out, i.e., the region where $d\sigma^0/d\tau < 0$ vanishes.

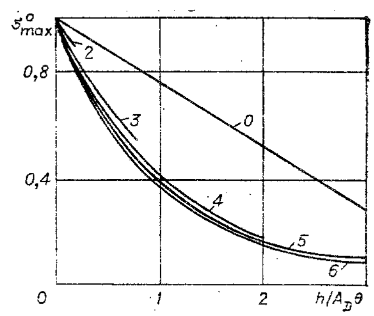

Fig. 4

But, even with x = 0.18, behind the shock wave at the front of the forerunner there is a further continuous rise in the stress. During this time, in the initial cross section the stress decreases only to 0.9997. The curves of $\sigma^0(\tau)$ (see Fig. 1a) relate to relatively great distances, where there is no longer a first maximum for the wave.

It follows from the calculations that the rate of propagation of the maximum of the stress D_m in a certain section rises with an increase in the distance. Under these circumstances, it tends toward a limit, depending on the value of $\mu\theta$. With small values of $\mu\theta$, the limiting velocity $D_m \sim 1$, i.e., it corresponds to the velocity of the forerunner, determined by the diagram of the dynamic compression. With an increase in $\mu\theta$. the limiting value D_m decreases. With $\mu\theta$ = 5, 50, 500, 50,000, D_m is equal, respectively, to 0.9, 0.7, 0.6, 0.5.

The rate of propagation of the maximum of the stress with small values of x is greater than the velocity of the maximum of the deformation and the maximum of the velocity of the particles. With a rise in $\mu\theta$, the values of these velocities approach each other. With $\mu\theta = 50$ and $x > 10$, the velocities practically coincide.

3. Scattering of Waves in Viscous Media and Deviation of the Parameters from the Similarity Condition

The calculated results given above show that viscous properties lead to scattering: with an increase in the duration of the wave, the rate of propagation of the maximum of the perturbation declines. With μ = 1000 sec^{-1}, this corresponds to soils of average density; with a rise in the duration of the loading θ, setting up the wave, from 0.005 to 0.05 sec, the velocity of the maximum decreases by approximately 1.3 times, and, with a change in θ from 0.05 to 0.5 sec, by 1.1 times.

The natural scatter in the properties of exactly the same soil leads, as experiments show [1], to a change in the velocity D_m by 1.3-1.5 times. Therefore, to observe scatter, the time of action of the wave must be varied by one and one-half to two orders of magnitude. With smaller intervals of the values of θ it is practically impossible to disclose the scatter, as is shown by the results of experiments. Let us compare the parameters of the waves in linear plastic and viscoplastic media. We use a model of a plastic medium in which the loading is linear $\sigma = E_D\varepsilon$ and unloading takes place along the straight lines $\sigma - \sigma_m = E_R(\varepsilon - \varepsilon_m)$ (σ_m is the maximal stress; ε_m is the maximal deformation in a particle). This model is a limiting case of the model used for a viscoplastic medium with $E_S \to E_D$ or $\mu \to 0$. For such a medium, an analytical solution has been obtained [1] to the problem of the propagation of a wave set up by shock loading, varying in the initial cross section according to Eqs. (5). The maximal dimensionless values of the stress, the deformation, and the velocity of the particles in the variables h, t are defined by the expressions

$$\sigma^0_{\max} = \varepsilon^0_{\max} = u^0_{\max} = 1 - \frac{A_R - A_D}{2A_R^2}\frac{h}{A_D\theta};$$

$$\sigma^0_{\max} = \frac{\sigma_{\max}}{\sigma_m};\quad \varepsilon^0_{\max} = \frac{\varepsilon_{\max}}{\varepsilon_m};\quad u^0_{\max} = \frac{u_{\max}}{u_m};$$

$$A_R = \sqrt{E_R\rho_0};\quad \frac{h}{A_D\theta} \leqslant \frac{A_R}{A_R - A_D};\quad A_D = \sqrt{E_D\rho_0}.$$

The maximal values of all three quantities coincide and vary in accordance with a linear law with the dimensionless distance $h/A_D\theta$.

Figure 4 gives curves of the dependence of the maximal stress in a wave on the dimensionless distance $h/A_D\theta = x/\mu\theta$ with different values of $\mu\theta$ in a viscoplastic (curves 2-6 correspond to variants 2-6 of

Table 1) and in a plastic medium (curve 0). The stress in a viscous medium varies in accordance with a nonlinear law; at all distances it is less than in a plastic medium. With different values of the parameter $\mu\theta$, the maximal values of the stress are close.

The calculations show that the maximal values of the deformation and the velocity of the particles in a viscous medium, with increasing distance from the initial cross section, vary according to a nonlinear law and, at close distances, exceed the values of these quantities in a plastic medium. With an increase in the distance, ε^0_{max} and u^0_{max} in a viscous medium decrease more rapidly and, at a sufficiently great distance, become smaller than in a plastic medium. In distinction from the stress, they vary appreciably with a change in $\mu\theta$. An increase in $\mu\theta$ by one or two orders of magnitude, at corresponding distances, leads to a change in ε^0_{max} and u^0_{max} by tenths of a precent, and to small changes in σ^0_{max}.

The deformation of a medium with the passage of a wave takes place with a variable deformation rate $\dot{\varepsilon}^0$. The distances where there are no shock waves, $\dot{\varepsilon}^0$ at first rises, attains a maximum, and then declines. Therefore, the sections of the diagram of $\sigma(\varepsilon)$ corresponding to a rise in the stress are found to be concave toward the axis of the deformations, even when the diagrams $\sigma = f(\varepsilon)$, corresponding to $\dot{\varepsilon} \to \infty$ and $\dot{\varepsilon} \to 0$, are linear or convex toward the axis of the deformations. After the load has been removed, residual deformations remain in the medium, whose value depends on the magnitude and the duration of the load setting up the wave.

4. Overall Character of Waves in Viscoplastic and Elastoplastic Media

In viscoplastic media, where the diagrams of the dynamic ($\dot{\varepsilon} \to \infty$) and static ($\dot{\varepsilon} \to 0$) compression are linear or convex with respect to the axis of the deformations, the character (profile) of the wave depends essentially on the distance from the initial cross section and on the value of the parameter $\mu\theta$. Under the action of unsteady-state shock loading, in the general case shock fronts exist only in the neighborhood of the initial cross section. Under these circumstances, there arises a double-wave configuration. A forerunner moves ahead, with a shock wave at the front, i.e., the first maximum. Behind the shock wave there is first a drop, and then a continuous rise in the stress up to a second maximum. After this, with increasing distance, there is a further gradual decrease in the magnitude of the shock wave at the forerunner down to zero. Under these circumstances, the maximal stress in the wave decreases, but still remains large. The time required for the stress to rise to a maximum increases. There arises a continuous compression wave which exhausts itself gradually with propagation.

In elastoplastic media, where the diagram of the compression with small stresses is assumed to be concave, and with large stresses, convex, with respect to the axis of the deformations, with the action of unsteady-state shock loading, there also arises a double-wave configuration [1, 8]. A forerunner moves ahead (an elastic wave), with a shock wave at the front. After this, there follows a region of constant flow (a plateau), along which the second shock wave, called a plastic wave, is propagated. The amplitude of the plastic wave decreases with increasing distance; under these circumstances, the duration of the region of constant flow rises. The amplitude of the forerunner remains unchanged up to the exhaustion of the plastic wave.

The experiments show that the picture of the propagation of waves in soils and rocks and in some other dense media is described most exactly by a model of a viscoplastic, and not an elastoplastic, medium. In actuality, the shock wave at the front of the forerunner falls rapidly to zero, where the maximal stress in the wave is still great. There is no plateau behind the front of the forerunner. The second maximum is washed out. The damping of waves in a viscous medium takes place more rapidly than in a plastic medium, which is closer to the experimental values.

The diagrams of compression with a variable sign of the curvature, taken as a basis in the model of an elastoplastic medium, as is shown by an analysis of experiments on the dynamic compression of samples, were obtained with a large, but finite, deformation rate, i.e., they do not relate to the limiting case $\dot{\varepsilon} \to \infty$. In addition, during the process of compression, the deformation rate $\dot{\varepsilon}$ was obviously not always held constant. If, with compression, in some interval of time $\dot{\varepsilon}$ decreases, the resulting curve of $\sigma(\varepsilon)$ is found to be concave, and, with a rise in $\dot{\varepsilon}$, convex toward the axis of the deformations, even if the dynamic diagram of the compression with $\dot{\varepsilon} \to \infty$ is linear.

Thus, the washing out of the waves with increasing distance [1, 6, 7] is connected with the viscous properties of the solid media, and not with a different sign of the curvature of the diagram of the dynamic compression with different values of the stress.

It is noted in [9] that the diagrams of the dynamic compression of soils and rocks are convex with respect to the axis of the deformations, and that shock waves exist only in the region of high stresses. Such a character of the change in the waves with increasing distance is explained by the dilatant properties of soils and rocks in a pulverized state (dilatance is the change in the volume of a granulated medium with shear).

Conditions of similarity with the propagation of waves are observed in a model of an elastoplastic medium and are not observed in a model of a viscoplastic medium. A large number of experiments show that the condition of similarity is satisfied in a first approximation in soils and rocks. The conclusions with respect to the satisfaction of the condition of similarity are based on a comparison of the values of the maximal stress in the wave with different values of its duration at identical dimensionless distances. The calculations carried out above show that in exactly the same viscous medium with different values of the duration of the loading, setting up the wave, at identical dimensionless distances $h/A_D\theta$, the maximal stresses differ only slightly. Therefore, observance of the condition of similarity with respect to the stress is not a very weighty argument in support of the use of a model of an elastoplastic medium. But the solution of wave problems taking account of viscous and plastic properties is complicated. Therefore, it is advisable, as before, to use the simpler model of an elastoplastic medium, which permits obtaining an approximate picture of the damping of the waves.

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